Approach for Evaluating Effects of Wall Losses on Quarter-Wave Short-Circuit Impedance Standards

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Abstract —The conservation of energy principle and first-order perturbation theory have been applied to obtain formulas for the physical lengths and reflection coefficient magnitudes of quarter-wave coaxial and rectangular waveguide short-circuit impedance standards. The expressions for the physical lengths ensure zero phase angle at the mating interface when wall losses are present.

The method can be extended to include small dielectric and magnetic losses, and requires only knowledge of the loss-free solutions. It can also be applied to other waveguiding structures which support uncoupled modes.

I. Introduction

THE QUARTER-WAVE short circuit was proposed [1] and developed [2] as an impedance standard for several coaxial and waveguide sizes about two decades ago. Several events have since suggested a re-evaluation of that earlier work. Programs such as MILSTAR have created a need for scalar and vector measurement capabilities at millimeter-wave frequencies. Indeed, an extension of the basic concepts and techniques to the WR-15 waveguide system has already been carried out [3]. The advent of high-performance vector network measurement systems such as the HP8510 have generated a need for improved calibration standards. There has also been recent interest in extending conventional coaxial transmission techniques to 60 GHz [4]. Taken together, these events suggest that improvements in microwave and millimeter-wave network measurements and techniques will be required both in coaxial and rectangular waveguide systems over an unprecedented range of frequencies.

Using quarter-wave short circuits as standards has several advantages. First, their characteristics can be well analyzed. Second, a relatively inaccurate knowledge of the conductivity of the coaxial or waveguide walls nevertheless yields an accurate reflection coefficient for the standard so long as the conductivity is high, as is the case in the most commonly used metals. For example, a 20-percent error in the conductivity of the copper walls will result in only a 0.015-percent error in the reflection coefficient of the standard [3]. Last, the standing-wave current minimum is located at the interface, thereby minimizing the effect of mating imperfections on performance. For these reasons, the quarter-wave short circuit may well be the best single-frequency network standard available. It should be noted that all the current millimeter-wave systems are also single-frequency systems.

In the following discussion, we have extended the previous work [1]–[3] to include the effects of wall losses on the phase angle of the reflection coefficient at the flange interface. The result is presented as a corrected line length for achieving a zero phase at the interface. We have also obtained a relation between the reflection coefficient magnitude and the corrected line length.

Our approach invokes the Poynting theorem, which furnishes an elegant method for deriving the properties of these standards.

II. Theory

The Poynting theorem is the starting point for evaluating the reflection coefficient of quarter-wave short-circuit impedance standards in the presence of wall losses. For harmonic fields with time variations of the form $e^{j\omega t}$, this theorem may be expressed as

$$\frac{1}{2}\oint_S \vec{E}\times\vec{H}^*\cdot(-d\vec{S}) = 2j\omega\int_V\left(\frac{\vec{B}\cdot\vec{H}^*}{4}-\frac{\vec{E}\cdot\vec{D}^*}{4}\right)dV + \frac{1}{2}\int_V \vec{E}\cdot\vec{J}^* dV. \quad (1)$$

Here, S is a closed surface that bounds volume V, and $d\vec{S}$ is the outward normal to the surface. If the volume V is filled with a medium whose permittivity $\epsilon=\epsilon'-j\epsilon''$, permeability $\mu=\mu'-j\mu''$, and conductivity σ_m are simple and nondispersive, and V is bounded by walls characterized by a surface impedance Z_m, defined by [5, p. 37.]

$$\vec{E}=Z_m\vec{J}_s,\quad Z_m=\frac{1+j}{\sigma\delta_s} \quad (2)$$

where the skin depth is given by

$$\delta_s=\sqrt{\frac{2}{\omega\mu\sigma}}$$

and where σ and μ denote the conductivity and permeability, respectively, of the walls, then (1) gives rise to the

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following relation:

$$\mathrm{Re}\,\frac{1}{2}\oint_S \vec{E}\times\vec{H}^*\cdot(-d\vec{S}) = \frac{\omega}{2}\int_V \left(\mu''\vec{H}\cdot\vec{H}\rightarrow * + \epsilon''\vec{E}\cdot\vec{E}^*\right)dV + \frac{1}{2}\int_V \sigma_m \vec{E}\cdot\vec{E}^*\,dV + \mathrm{Re}\,\frac{1}{2}\int_{\text{walls}} Z_m \vec{J}_s\cdot\vec{J}_s^*\,dS \quad (3)$$

for the real part, and

$$\mathrm{Im}\,\frac{1}{2}\oint_S \vec{E}\times\vec{H}^*\cdot(-d\vec{S}) = 2\omega\int_V\left(\frac{\mu'\vec{H}\cdot\vec{H}^*}{4} - \frac{\epsilon'\vec{E}\cdot\vec{E}^*}{4}\right)dV + \mathrm{Im}\,\frac{1}{2}\int_{\text{walls}} Z_m \vec{J}_s\cdot\vec{J}_s^*\,dS$$
$$= 2\omega(W_m - W_e) + \mathrm{Im}\,\frac{1}{2}\int_{\text{walls}} Z_m \vec{H}\cdot\vec{H}^*\,dS \quad (4)$$

for the imaginary part. Here, $\vec{J}_s$ is a surface current on the walls and is related to the tangential magnetic field at the surface by

$$\vec{J}_s = \hat{n}\times\vec{H}$$

where $\hat{n}$ is a unit outward normal to the surface, and W_m and W_e are the magnetic and electric energy, respectively, stored in V.

Equations (3) and (4) differ from their counterparts in more conventional analyses [5, p. 32], in that the contributions to the loss in the system arising from the bulk conductivity σ_m of the material filling volume V, and those due to the finite conductivity of the walls, have been separated and displayed explicitly. Due to the complex nature of Z_m, such a separation results in an additional energy storage term given by the second integral on the right side of (4). This term represents the energy stored in the fields associated with the current distribution $\vec{J}_s$ flowing in the walls. In our analysis, σ_m will be assumed negligible since the medium filling the volume of interest usually is air.

Equation (3) forms the basis for computing the perturbations to the magnitude of the reflection coefficient caused by wall losses. It can also be used to evaluate the effects of dielectric and magnetic losses, although that is not done here.

Similarly, (4) can be applied to obtain the phase angle perturbations in the reflection coefficient due to wall losses. The stored electric and magnetic energy can conveniently be expressed in terms of the incident power P_{inc} for both the coaxial and rectangular waveguides. The first term on the right side of (4) is proportional to the excess of the magnetic over the electric energy stored in the volume. For the coaxial line, as well as for the rectangular waveguide, both of length l, this excess stored magnetic energy is

$$2\omega(W_m - W_e) = 2P_{\text{inc}}\sin\theta \quad (5)$$

where $\theta = 2\beta l$ is twice the electrical length of the short circuit. This expression in conjunction with (4) readily shows that for the lossless case ($Z_m = 0$), the requirement that the net stored energy equals zero at resonance implies that $\theta = \pi$, or that the length l equals exactly a quarter wave or integral multiple thereof. When the walls have finite conductivity $Z_m \neq 0$, and the resonance condition obtained from (4) is now

$$\sin\theta + \frac{P_l}{2P_{\text{inc}}} = 0. \quad (6)$$

Here, P_l arises from the energy storage associated with the wall currents and, since Z_m has equal real and imaginary parts as displayed in (2), numerically equals the power loss in the walls. Equation (6) shows that wall losses cause the length of the short circuit to deviate from a quarter wave. Since P_l and P_{inc} are positive, θ lies in the third quadrant, i.e., the corrected length is somewhat larger than a quarter wavelength due to the magnetic energy stored in the walls. Note also that (6) has been obtained without regard to any waveguide or transmission-line cross section; therefore, it has applicability to a large class of nonradiating waveguiding systems. Furthermore, it can be modified to incorporate open transmission line and/or waveguiding media provided that the loss terms associated with surface-wave leakage and radiation can be computed and included into the power loss. Lastly, (6) forms the basis for computing phase error in the short-circuit standard.

For small losses, θ will exceed π by a small amount $\Delta\theta$ so that $\sin\theta \cong -\Delta\theta$. Then, (6) may be expressed in terms of the correction to the electrical length as

$$\Delta\theta = \frac{P_l}{2P_{\text{inc}}}. \quad (7)$$

The surface integral on the left side of (3) yields the difference between the incident and reflected power at the connector interface, while the right side represents the losses in the volume and the walls. Thus, (3) is a statement of conservation of energy within the one-port network. Since air is the most commonly used dielectric medium, we may neglect its losses at most frequencies of interest and consider only the wall losses. The magnitude of the reflection coefficient is given by

$$P_l/P_{\text{inc}} = 1 - |\Gamma|^2. \quad (8)$$

Again, assuming small losses, the magnitude of the reflection coefficient can be approximated well by the leading term in the binomial expansion to give

$$|\Gamma| = 1 - \Delta\theta \quad (9)$$

where $\Delta\theta$ is given in (7).

The calculation method consists of using the loss-free solutions to calculate the incident power and power loss for the length l of the coaxial line or waveguide and evaluating the correction $\Delta\theta$ from (7). The length of the quarter-wave short-circuit standard can then be calculated from

$$l = \frac{\lambda}{4} + \Delta l = \frac{\lambda}{4} + \frac{\Delta\theta}{2\beta} \quad (10)$$

where λ is the appropriate guide wavelength and β is the corresponding propagation constant for the loss-free situation. Eliminating $\Delta\theta$ between (9) and (10) leads to an expression for the reflection coefficient magnitude in terms of l, namely,

$$|\Gamma| = 1 - \pi\left(\frac{4l}{\lambda} - 1\right). \tag{11}$$

We now illustrate the method for the case of the coaxial TEM line and the rectangular waveguide.

III. Applications

A. Coaxial TEM Line

Fig. 1 is a view of the coaxial short circuit sectioned by a plane that contains the cylinder diameter and the direction of propagation z. The radius of the inner cylinder is a; the inside radius of the outer cylinder is b. The guiding structure terminates in a conducting plane normal to the direction of propagation of the wave and located at $z = 0$. We are assuming implicitly that the cylinders and the terminal plane are made of the same material, and that the space enclosed by the conductors is filled with a dielectric (usually air). The connector interface is shown at $z = -l$. For this geometry, we calculate the power loss in terms of the incident power as

$$P_l = P_{\text{inc}}\left\{\frac{\delta_s\left(\frac{1}{a}+\frac{1}{b}\right)}{2\ln(b/a)}(\theta + \sin\theta) + 2\beta\delta_s\right\} \tag{12}$$

where $\beta = \omega\sqrt{\epsilon\mu}$ is the magnitude of the propagation vector for the TEM wave. The first term in (12) arises from the coaxial walls, and the second term is due to the finite nonzero impedance of the short circuit. Substituting this result into (7) gives

$$\Delta\theta = \beta\delta_s\left\{\frac{\lambda}{8}\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{\ln(b/a)} + 1\right\} \tag{13}$$

which can be substituted into (10) to give the corrected line length

$$l = \frac{\lambda}{4} + \Delta l = \frac{\lambda}{4} + \frac{\delta_s}{2}\left\{\frac{\lambda}{8}\frac{\left(\frac{1}{a}+\frac{1}{b}\right)}{\ln(b/a)} + 1\right\}. \tag{14}$$

Then, (11) can be used to compute the reflection coefficient magnitude. For example, if $f = 4$ GHz ($\lambda = 7.495$ cm), $b = 0.35$ cm, $a = 0.152$ cm, and $\sigma = 10^5$ S/cm, then $\delta_s = 2.516$ μm and (14) gives $l = 1.8751$ cm, which exceeds a quarter wavelength by 0.0014 cm or approximately 4.21 min. of arc. Substituting the corrected length in (11) gives $|\Gamma| = 0.99755$, which corresponds to a return loss of 0.0213 dB and agrees with previous computations [2].

B. Rectangular Waveguide

Similar conclusions can be drawn about the dominant TE mode propagating toward a conducting plane parallel

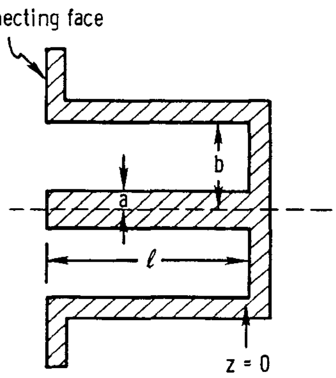

Fig. 1. Schematic sectional view of a coaxial quarter-wave short circuit, illustrating the relevant dimensions used in the text.

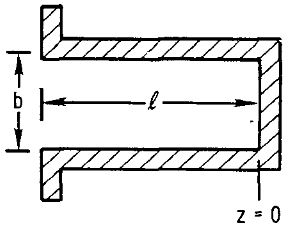

Fig. 2. E-plane section of a rectangular waveguide quarter-wave short circuit. l is slightly larger than a quarter of a guide wavelength.

to the cross section of a rectangular waveguide of width a and height b and located at $z = 0$. Fig. 2 illustrates the geometry for this case. The power loss is calculated in terms of the incident power and the propagation constant β of the waveguide mode as follows:

$$P_l = P_{\text{inc}}\frac{\delta_s}{b}\left\{\left(\frac{\pi}{\beta a}\right)^2\left(1+\frac{2b}{a}\right)(\theta - \sin\theta) + \theta + \sin\theta + 2\beta b\right\} \tag{15}$$

which gives the correction to the electrical length

$$\Delta\theta = \beta\delta_s\left\{\frac{1 + \frac{\lambda_g}{4b}(1+L)}{1 - \frac{\delta_s}{b}L}\right\} \tag{16}$$

where L is defined for convenience as

$$L = \left(1 + \frac{2b}{a}\right)\frac{(f_c/f)^2}{\epsilon_r - (f_c/f)^2}. \tag{17}$$

Here, again, we have accounted for the finite nonzero impedance of the shorting plate. Proceeding as for the coaxial TEM line, we obtain the corrected length

$$l = \frac{\lambda_g}{4} + \frac{\delta_s}{2}\left\{\frac{1 + \frac{\lambda_g}{4b}(1+L)}{1 - \frac{\delta_s}{b}L}\right\}. \tag{18}$$

For a WR-90 copper waveguide, where $a = 2.286$ cm, $b = 1.016$ cm, and $f_c = 6.557$ GHz, we have at 9.4 GHz, $\delta_s = 0.6816$ μm and $\lambda_g = 4.4511$ cm. These values give

$l = 1.1129$ cm, which exceeds a quarter of a guide wavelength by 1.383 μm. From (11), we now obtain $|\Gamma| = 0.9996$, which corresponds to a return loss of 0.0034 dB and again agrees well with previous computations [2]. Note that the guide wavelength must be substituted into (11) to give $|\Gamma|$.

IV. Summary

We have demonstrated an application whereby the Poynting theorem is used to evaluate the effects of wall losses on coaxial and rectangular transmission-line quarter-wave short-circuits. The results show that the physical line length must be slightly longer than previously reported [3] to ensure zero phase or no current flow at the flange interface. Due to this, the magnitude of the reflection coefficient is immune to mating imperfections. Other loss mechanisms, such as dielectric loss, can also be included provided they are small enough to conform with the usual requirements of perturbation methods. Additionally, the principles are readily extended to other waveguiding systems once the loss-free solutions in those systems are known.

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